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Feedback control of a masonry vault

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Abstract

Ancient buildings need sometimes the supply of reinforcing structures; *smart* linkage systems between old and new timbers can then provide for the necessary flexibility in view of partly indeterminate load conditions. The design of active reinforcements requires, in particular, the choice of a control policy. Predictions based on available mathematical models being not reliable enough, output feedback generated controls must eventually be used. Problems of non-linearity and of controllability, due to control constraints, arise.

The vault of the *Maggior Consiglio* Hall in the Duke's Palace of Genoa, though presently supported by an auxiliary truss, undergoes relatively large displacements probably due to temperature variations; the use of stiffer links to reduce these displacements may result into high thermal stresses. This problem will be studied here as an example to design a control system. A numerical model of the structure is proposed, and a performance function is considered; a feedback loop is designed and the corresponding algorithm is given. Results show the feasibility of the method.

1 Foreword

The structural restoration of masonry buildings often demands the introduction of auxiliary structures; the linkage between the old and the new fabric must then be designed to avoid unwanted constraints, stress concentrations which would localise damage, etc. Active auxiliary structures can answer to the need of ready adaptation to predictable, but poorly determinable, events, supplying the old structure with strength or ductility where and when needed. They require the definition of an appropriate control policy and the design of a fit control system to implement the strategy.

Feedforward algorithms may be hindered, in practice, due to the relatively poor power of estimation of available mathematical models of complex masonry structures and to the lack of data on the contexture and constitution of materials within foundations, walls, vaults, etc. Consequently, as in most engineering applications, feedback controls are needed.

Non-linearity of material or structural behaviour is in general to be expected, especially if—as in seismic circumstances—the ancient structure's strength resources must be wholly exploited. A proficient mathematical model of the structure must then anyway be included in the control system, in order to predict the effects of application of the feedback generated control actions. These actions must then be constrained to avoid damaging the ancient structure, and the controllability of the system must be analysed.

2 Introduction

The vault of the *Maggior Consiglio* Hall in the Duke's Palace of Genoa was built shortly after a fire destroyed the first floor of the Palace in 1777. To reduce the vulnerability to future events of the kind, Simone Cantoni, the architect, conceived an entirely masonry structure to cover the 17 times 35 meters surface of the greater hall of the Palace; due to the wide span and, probably, to appearance requirements two superposed vaults were built.

Some unexpected event, perhaps a settlement of the foundation or of the underlying medieval structure, or some technical error, caused the inner vault to undergo localised distortions already during building in 1783. The thrust of the arches of the outer vault on the wall of the southern facade of the Palace was then partly reduced by four tie rods.

Presuming this thrust be the main cause of the disease, the heavy masonry covering was removed in 1861 by architect Ignazio Gardella, and a roof supported by iron trusses was built in its place. Each of the four damaged arches of the remaining inner vault was tied with rods to the truss above. Also the facade was reinforced with buttresses formed into two orders of columns. Notwithstanding improvements, the degradation of the vault went not to a stop.

During the last restoration of the Palace, finished in 1992, steps were taken also to reduce or eliminate the structural weakness of the vault. The walls supporting it were supplied with a concrete ring-beam at the eaves level and reinforced through injections and micro-piles; auxiliary iron truss-beam were built between the vault and the roof and connected to the vault to support it through a number of springs (cfr. Osti [1] and Croci [2]).

Though the risk of failure is presently reasonably reduced, movements of the vault can still be observed and seem to cause, at least, damage to the ornaments and frescos of the magnificent ceiling. A measurement campaign carried out by the authors of the project showed that a correlation exists between these movements and temperature. Obviously, mainly vertical displacements result from thermal dilatations of the vault due to its lateral confinement. Though slow phenomena are involved, the cyclic nature of the thermal load must be considered as particularly

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critical for the masonry structure. Some of the mortar joints in the most stressed regions of the structure may undergo cycles of opening/closing of micro-cracks (when not of macroscopic fractures) to accomplish the overall deformation. In these conditions the stability of cracks propagation need be verified.

To reduce the risk of increasing damage one should wish to confine the amplitude of cyclic displacements due to the thermal load. With a standard, passive, reinforcement, the only possible way to achieve this result is to constrain even further the displacements, e.g., linking the vault to the auxiliary structure with stiffer springs or even through rigid rods. A drawback of this strategy (which was certainly discarded by the authors of the restauration project) is that it increases the thermal stresses and eventually rises more problems than it solves.

A smarter approach would be to replace the springs currently linking the auxiliary structure to the vault with motion controllers and implement an appropriate control strategy of the system. The target to pursue could be a minimum deviation of the displacement of the vault from some ideal conditions (say, the static deflections under permanent load at uniform temperature, the latter being chosen, e.g., as the mean temperature in the season).

In [3] a FEM model of the vault was put forward and some results of computations under extreme thermal conditions (coldest and hottest possible conditions for the site) were shown. Notice the possible effect, in summer, of the radiation of heat on the black tiles of the roof and of the air conditioning of the hall.

In this paper we study the feasibility of a feedback control on the vault under external thermal cyclic loads using active bonds to link the vault and the truss. The output measures needed to feed the control loop are obtained through the quoted FEM model of the vault, taking a simulated temperature variations from winter to summer conditions. The same model was used to evaluate, at each step of loading, the correlation matrix governing the control.

Here we define the operating range of actuators and a feedback control algorithm building the correlation matrix at each step of loading. Our results support the feasibility of the control.

3 Description of the system

Call $\mathbf{u}$ the list of degrees of freedom, $\mathbf{K}$ the stiffness matrix, $\mathbf{f}$ the generalised forces acting on the structure. Consider a partition of the degrees of freedom, use index u to denote the degrees on which the controls act, x for the degrees which must be controlled, and s for all the others, part of which belong to the boundary. Furthermore let $\mathbf{f}^{(\theta)}$ be a particular thermal load condition, $\mathbf{f}^{(\gamma)}$ the permanent loads, and $\mathbf{f}_u^{(\kappa)}$ a particular set of forces applied on the structure by the actuators; by assumption it is $\mathbf{f}_x^{(\kappa)} = 0$ and $\mathbf{f}_s^{(\kappa)} = 0$. To compact notations we will denote combined loads with sum of exponents:

$$\mathbf{f} = \mathbf{f}^{(\theta)} + \mathbf{f}^{(\gamma)} + \mathbf{f}^{(\kappa)} = \mathbf{f}^{(\theta+\gamma+\kappa)} . \tag{1}$$

As already mentioned there are technical limits to the force that can be impressed on the structure through the actuators. Call $\bar{f}$ the absolute value of such a

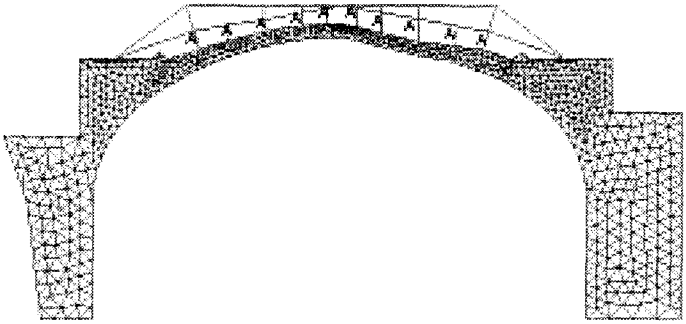

Figure 1: Finite element mesh of the structure; an A indicates each force actuator.

limit on a single linkage (here for shortness we assume all linkages working below a force of $10kN$ in the positive and negative directions); each control force must verify the conditions:

$$-\bar{f} \leq f_{u\,i} \leq \bar{f}\,, \tag{2}$$

where a_i denotes the i-th entry of the array $\mathbf{a}$ and clearly $\mathbf{f}_u = \mathbf{f}_u^{(\theta+\gamma+\kappa)}$ (notice that, the actuators being relatively compliant, the first two contributions to the whole force ought to be significantly smaller than the third; $|f_{u_i}^{(\theta+\gamma)}| < \bar{f}$ largely enough).

Temperature variations being rather slow, it is reasonable to assume that prompt control reactions will anyway be slow enough not to rise (undesirable) dynamic effects on the structure; then the description of the system can be done neglecting inertia forces. Furthermore non linear effects, if any, are also assumed here to be time independent (e.g., quasi-static opening and closing of cracks, deformation induced damage, etc.).

4 Control algorithm

A simple strategy would be that of minimising at all time (time is now just a parameter ordering events) the absolute value of displacements of type 'x' ($\mathbf{K}$ depends on the process):

$$\begin{cases} \mathbf{K}\mathbf{u} + \mathbf{f} = 0\,, \\ \underset{\mathbf{f}_u^{(\kappa)}}{\mathrm{Min}}\ \{|\mathbf{u}_x| \quad | \quad |f_{u\,i}| \leq \bar{f}\ \forall i\} \end{cases} \tag{3}$$

(plus boundary conditions). Call $\tilde{\mathbf{K}}$ the tangential stiffness matrix. If, the present

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displacements being $\mathbf{u}$, the thermal loads increase of $\Delta\mathbf{f}^{(\theta)}$, the answer of the control systems in terms of an increase of control forces $\Delta\mathbf{f}_u^{(\kappa)}$ will be the solution of:

$$\begin{cases} \tilde{\mathbf{K}}\mathbf{v}^{(\theta)} + \Delta\mathbf{f}^{(\theta)} = 0\,, \\ \tilde{\mathbf{K}}\mathbf{v}^{(\kappa)} + \Delta\mathbf{f}^{(\kappa)} = 0\,, \\ \underset{\Delta\mathbf{f}_u^{(\kappa)}}{\mathrm{Min}} \left\{ |\mathbf{u}_x + \mathbf{v}_x^{(\theta)} + \mathbf{v}_x^{(\kappa)}| \;\; \middle| \;\; |f_{ui}| \leq \bar{f}\ \forall i \right\}. \end{cases} \tag{4}$$

The stiffness matrix can be partitioned as:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{ux} & \mathbf{K}_{us} \\ \mathbf{K}_{ux}^T & \mathbf{K}_{xx} & \mathbf{K}_{xs} \\ \mathbf{K}_{us}^T & \mathbf{K}_{xs}^T & \mathbf{K}_{ss} \end{bmatrix}, \tag{5}$$

calling

$$\begin{aligned} \begin{bmatrix} \mathbf{\Phi}_{xu} \\ \mathbf{\Phi}_{su} \end{bmatrix} &= \begin{bmatrix} \tilde{\mathbf{K}}_{xx} & \tilde{\mathbf{K}}_{xs} \\ \tilde{\mathbf{K}}_{xs}^T & \tilde{\mathbf{K}}_{ss} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{K}}_{ux}^T \\ \tilde{\mathbf{K}}_{us}^T \end{bmatrix}, \\ \mathbf{H} &= \tilde{\mathbf{K}}_{uu} - \begin{bmatrix} \tilde{\mathbf{K}}_{ux} & \tilde{\mathbf{K}}_{us} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{xu} \\ \mathbf{\Phi}_{su} \end{bmatrix}, \end{aligned} \tag{6}$$

the increment of displacement due to the control forces can be searched solving the condensed system:

$$\begin{cases} \mathbf{H}\mathbf{v}_u^{(\kappa)} + \Delta\mathbf{f}_u^{(\kappa)} = 0\,, \\ \mathbf{v}_x^{(\kappa)} = -\mathbf{\Phi}_{xu}\mathbf{v}_u^{(\kappa)}\,, \\ \mathbf{v}_s^{(\kappa)} = -\mathbf{\Phi}_{su}\mathbf{v}_u^{(\kappa)}\,. \end{cases} \tag{7}$$

Formally the control problem is reduced to the search of:

$$\underset{\Delta\mathbf{f}_u^{(\kappa)}}{\mathrm{Min}} \left\{ |\mathbf{u}_x + \mathbf{v}_x^{(\theta)} + \mathbf{S}_{xu}\Delta\mathbf{f}_u^{(\kappa)}| \;\; \middle| \;\; |f_{ui}| \leq \bar{f}\ \forall i \right\}, \tag{8}$$

where $\mathbf{S}_{xu} = \mathbf{\Phi}_{xu}\mathbf{H}^{-1}$ and $\mathbf{v}^{(\theta)}$ is the solution of

$$\tilde{\mathbf{K}}\mathbf{v}^{(\theta)} + \Delta\mathbf{f}^{(\theta)} = 0\,. \tag{9}$$

In practical circumstances of feedback loops, the matrix $\mathbf{S}_{xu}$ and the displacements $\mathbf{u}_x + \mathbf{v}_x^{(\theta)}$ can be evaluated by direct measure. $\mathbf{S}_{xu}$ is the compliance correlation matrix giving generalised displacements of type x corresponding to unit forces of type u; its n-th column is the list of $\mathbf{u}_x^{(\tau)}$ displacements corresponding to application of test forces $f_{u\,n}^{(\tau)} = 1$, all other entries of $\mathbf{f}_u^{(\tau)}$ being null; sampling can be automatically performed on the structure when needed (e.g. at each relevant change of measured displacements) using the actuators to apply the test forces. The controlled degrees of freedom x should be supplied with sensors to monitor the actual state of the structure.

5 Implementation of the algorithm

The control algorithm described in the previous section is implemented through the steps of the following loop:

1. simulate a sample $\mathbf{u}_x$,
2. simulate an evaluation of $\mathbf{S}_{xu}$, i.e. do for all i:
   (a) $f_{u_i}^{(\tau i)} = \delta_{ij}$,
   (b) simulate application of $\mathbf{f}_u^{(\tau i)}$ on the structure,
   (c) simulate measurement of $\mathrm{col}_i(\mathbf{S}_{xu}) = \mathbf{u}_x^{(\tau i)}$,
3. solve minimisation (8) with a constrained linear least square algorithm,
4. simulate application of the increment $\Delta\mathbf{f}_u^{(\kappa)}$ on the structure,
5. simulate measurement of $\mathbf{u}_x$,
6. compare the present and previous entries of $\mathbf{u}_x$ and:
   (a) go to line 1 if they are close enough,
   (b) go to line 3 if there is a sensible difference,
   (c) go to line 2 if there is a large difference.

If an open loop control method is sought the algorithm is simpler (but care must be taken for the accuracy of predictions needed at step 2):

1. simulate a sample of temperature,
2. evaluate $\mathbf{u}_x$,
3. simulate an evaluation of $\mathbf{S}_{xu}$ (see the corresponding closed loop steps),
4. solve (8) with a LSQ algorithm,
5. simulate application of the increment $\Delta\mathbf{f}_u^{(\kappa)}$ on the structure,
6. go to line 1.

Two different targets of the control were considered, corresponding to different sets of points 'x':

- 10 points of the extrados of the vault on which the motion controllers act ('u' and 'x' degrees of freedom coincide; strategy 1),
- 10 points of the intrados where the permanent deformations are larger (strategy 2).

Correspondingly two different monitoring conditions for feedback must be implemented.

Numerical results have been obtained for a simulated history of thermal loads. In figure 3 we plot the results for extreme winter conditions, if strategy 1 is applied; open and closed loop results are practically equivalent. In figure 4 the same kind of results are shown, considering extreme summer conditions; in this case there is a little discrepancy between the two implemented methods.

In figure 5 we show the uncontrolled displacements (vertical components) vs time, considering a continuous variation of temperature during time from winter to summer conditions. The closed loop controlled displacements for strategy 2 are shown in 5.

The analysis of our results shows the feasibility and efficacy of the proposed control system.

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References

[1] Osti, C., Oliva, C., Perego, F., Spalla, G., Croci, G., Cerone, M., *Palazzo Ducale*, Editer: Roma, 1988.

[2] Croci, G., *Il Palazzo Ducale di Genova*, doc. Ente Palazzo Ducale, 1996.

[3] Brocato, M., Zanghì, F. Numerical evaluation of the synthesis control function for an actively reinforced masonry vault. *Proc. 1th Int. Conf. Albert Caquot: Modelling and Simulation in Civil Engineering, Paris, 3-5 Oct. 2001*, under press.

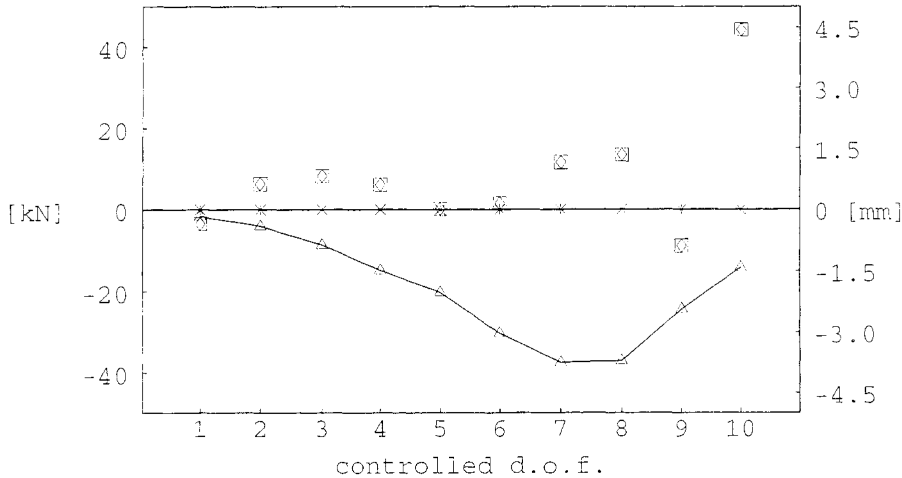

Figure 2: Strategy 1.: plot of uncontrolled (triangle), open loop controlled (x), and closed loop controlled (+) vertical displacements [mm], and of open loop (diamond) and closed loop (square) control forces [kN] vs a coordinate taken along the vault's cross section for extreme winter conditions.

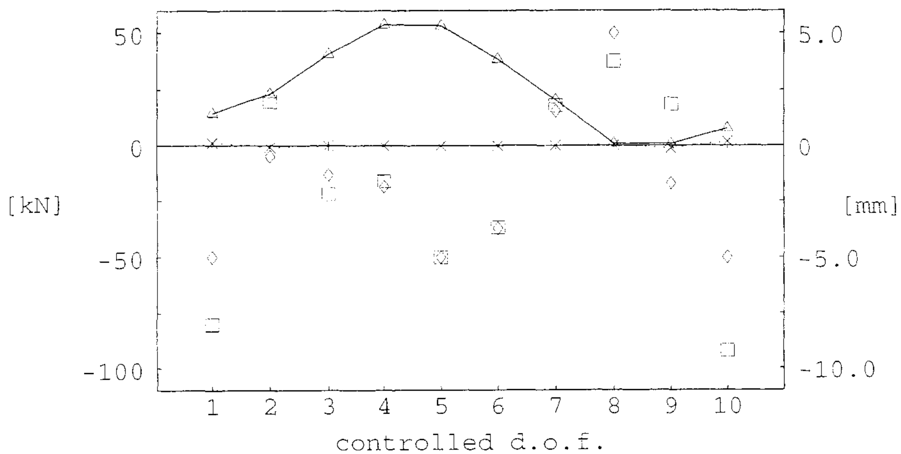

Figure 3: Strategy 1: same as figure 2 for extreme summer conditions.

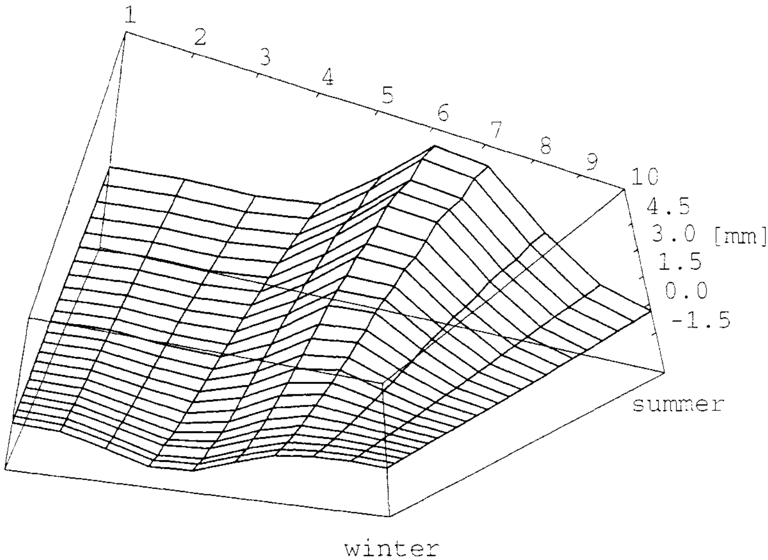

Figure 4: Plot of the uncontrolled vertical displacement of the intrados of the damaged part of the vault vs time simulating a variation of temperature conditions from winter to summer.

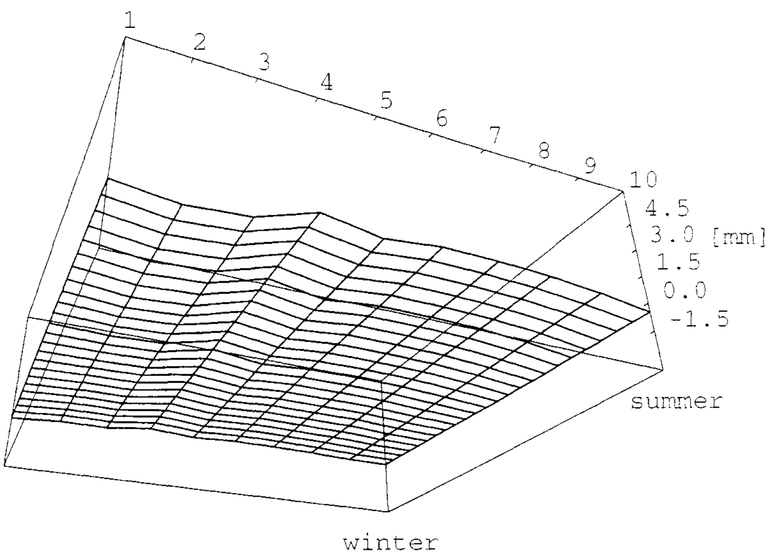

Figure 5: Strategy 2: plot as in figure 4 of the feedback controlled vertical displacements.